Model of nonlinear kinetic Alfvén waves with dissipation and acceleration of energetic electrons

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The acceleration of energetic electrons has long been one of the most outstanding problems in astrophysics and space physics, and some recent observations from space satellites show that low-frequency electromagnetic fluctuations in the auroral ionosphere and magnetosphere can often be identified as the kinetic Alfvén modes. A model of nonlinear kinetic Alfvén waves is presented here, in which the effect of electron collisional dissipation has been taken into account. The result is a *dissipative* solitary kinetic Alfvén wave (DSKAW), which can produce a local shocklike structure with a net electric potential drop and which can thereby accelerate efficiently the electrons to the order of the local Alfvén velocity. Since Alfvénic fluctuation is the most common electromagnetic activity in extensive cosmic plasma environments, the present result suggests that the DSKAW could play an important role in the acceleration and energization of cosmic plasmas.

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The production of energetic particles is very common in extensive cosmic plasma environments from auroral electrons in the magnetosphere and energetic particles in solar flares to energetic cosmic rays. The physical mechanism of the acceleration of these energetic particles, however, has long been one of the most outstanding problems in astrophysics and space physics. It is, however, still an open problem as to how the acceleration takes place. On the other hand, intense Alfvénic fluctuation is the most common electromagnetic activity in cosmic plasmas. In particular, some recent studies of *in situ* observations by space satellites on the polar orbits have shown that the physical nature of the strong electric spikes in the auroral ionosphere and magnetosphere can be explained in terms of a solitary kinetic Alfvén wave (SKAW) because (1) the ratio of the perturbed electric to magnetic fields, $\Delta E/\Delta B \sim v_A$ (the local Alfvén velocity); (2) the perpendicular scale size $\lambda_\perp \sim$ several λ_e (the local electron inertial length); and (3) these spikes are frequently accompanied with strong density fluctuations ($dn/n \sim 50\%$) [1–7]. A kinetic Alfvén wave (KAW) can be created when obliquely propagating shear Alfvén waves are affected by the ion gyroradius (for $\beta > Q$), or by the electron inertia length (for $\beta < Q$) such that a nonzero parallel electric field arises within the wave itself [8,9], and the existence of the SKAWs has been investigated theoretically under various conditions of plasma parameters by many authors [10–14] since the pioneering theoretical work of Hasegawa and Mima in 1976 [15], where β is the ratio of thermal to magnetic pressures, $Q \equiv m_e/m_i$ is the mass ratio of electrons to ions.

The possibility of linear KAWs accelerating and heating plasmas has been an increasingly interesting topic for discussion extensively in the fields of laboratory [16], space [17], and astrophysical plasmas [18] because of the capability of their parallel electric fields accelerating charged particles as well as the most common Alfvénic fluctuation in cosmic plasmas. However, the possible role of nonlinear KAWs (especially SKAWs) in the accelerating and heating plasmas has

not been discussed yet because there is no net electric potential drop over them due to the symmetry in their structures [19]. However, some further analyses of data revealed clearly that electron collisional dissipation could considerably affect the structure and evolution of the SKAWs. For example, on the basis of the analysis for about 100 SKAW events observed by FREJA, Wahlund *et al.* [20] these events could be classified as three different observational phases, and they were possibly responsible for three different stages in the dynamical evolution of the SKAWs due to the effect of the dissipation caused by the electron collision with turbulent ion acoustic waves [20]. In this paper, we present a model of nonlinear KAWs, in which the effect of electron collisional dissipation will be taken into account. The result is a *dissipative* solitary kinetic Alfvén wave (DSKAW), which can produce a local shocklike structure with a net electric potential drop and which can thereby accelerate efficiently the electrons to the order of the local Alfvén velocity. In particular, this suggests that the DSKAW could play an important role in the acceleration and energization of cosmic plasmas in the common Alfvénic fluctuations.

The equations governing the dynamics of the DSKAWs propagating in the x-z plane in a low-β plasma magnetized by a homogeneous ambient magnetic field $\mathbf{B}_0$ along the z direction can be written as follows [11–13]:

$$\partial_t n_e + \partial_z(n_e v_{ez}) = 0, \tag{1}$$

$$\partial_t n_i + (\omega_{ci}B_0)^{-1}\partial_x(n_i\partial_t E_x) = 0, \tag{2}$$

$$(\partial_t + v_{ez}\partial_z)v_{ez} = -(e/m_e)E_z - \nu_e v_{ez}, \tag{3}$$

$$\partial_z E_x - \partial_x E_z = -\partial_t B_y, \tag{4}$$

$$\partial_x B_y = -\mu_0 e n_e v_{ez}, \tag{5}$$

$$n_i \simeq n_e = n, \tag{6}$$

where ν_e is the collisional frequency of electrons with ions or turbulent waves, ∂ represents the partial derivatives with its subscripts, and the other symbols have their usual mean-

ings. In the above equations we have assumed (i) the low-β condition $\beta \ll Q$, so that the thermal pressure may be neglected in Eq. (3), (ii) the low-frequency limitation $\partial_t \ll \omega_{ci}$, so that the x component of the ion velocity may be approximated by the polarization drift velocity in Eq. (2), and (iii) the quasineutrality approximation $n_i \approx n_e$ in Eq. (6). In addition, the momentum equation of ions has been neglected because of $v_{iz} \sim Q v_{ez} \ll v_{ez}$.

In the wave frame defined in the following form:

$$\eta = k_x x + k_z z - M t, \tag{7}$$

taking the unperturbed boundary conditions as

$$\eta \to +\infty \Rightarrow n = n_0, v_{ez} = 0, \partial_\eta n = 0, \tag{8}$$

the nonlinear equation governing the density behavior of the DSKAWs can be derived from Eqs. (1)–(6) as

$$d_\eta^2 n = 3 n^{-1} (d_\eta n)^2 + k_x^{-2} n^2 (1-n)(M_z^{-2} - n) + \gamma n d_\eta n, \tag{9}$$

and the parallel components of the electron velocity v_{ez} and the perturbed electric field E_z are

$$v_{ez} = -(n^{-1} - 1) M_z \tag{10}$$

and

$$E_z = k_z M_z^2 [n^{-3} d_\eta n + \gamma (n^{-1} - 1)], \tag{11}$$

respectively, where

$$\gamma \equiv \sqrt{\frac{Q}{M}} \frac{\nu_e}{\omega_{ci}} \tag{12}$$

is the damping coefficient determined by the collisional frequency ν_e, $k_x = \sin\theta$, $k_z = \cos\theta$, $M_z \equiv M/k_z$, and θ and M are the angles between the wave propagating direction and the ambient magnetic field and the phase speed of the wave in the units of the Alfvén velocity v_A, respectively. In the above equations, space, time, density, velocity, and electric and magnetic fields have been normalized to λ_e, λ_e/v_A, n_0, v_A, $\sqrt{Q} v_A B_0$, and $\sqrt{Q} B_0$, respectively.

To restore the dimension, the damping coefficient γ in Eq. (12) can be rewritten as

$$\gamma = (\nu_e/\omega) k \lambda_e, \tag{13}$$

where $\omega = M v_A k$ is the frequency of the wave. For the KAWs in a low-β plasma, one has $k\lambda_e \sim O(1)$, and hence $\gamma \sim \nu_e/\omega$. Neglecting the dissipative effect (i.e., taking $\gamma = 0$), Wu *et al.* [19] derived an exact analytical solution of Eq. (9), which describes the density distribution of the SKAWs [see Eq. (22) and Fig. 2 in Ref. [19]]. For the case of the DSKAWs with $\gamma \neq 0$, we can obtain numerically the solution of Eq. (9). The solution with the parameters $\gamma = 0.1$, $M_z = 1.29$, and $k_x = \sin 89°$ is illustrated in Fig. 1, where Figs. 1(a)–1(c) are the distributions of the density (n), the parallel electric field E_z, and the velocity of electrons (v_{ez}), respectively. For the sake of comparison, the solution of the
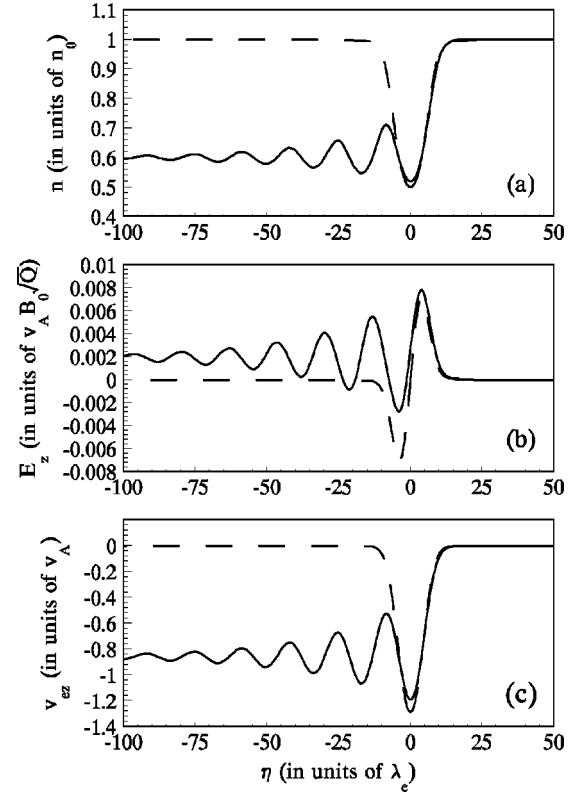

FIG. 1. Sketch of the shocklike structure of the DSKAW for the parameters $\gamma = 0.1$, $M_z = 1.29$, and $\theta = 89°$: (a) the density n; (b) the parallel electric field E_z; and (c) the parallel velocity of electrons v_{ez}, where the dashed lines represent the corresponding solution of the SKAW with the same M_z and θ, but $\gamma = 0$ (see Ref. [16]).

corresponding SKAW with the same $M_z = 1.29$ and $k_x = \sin 89°$, but $\gamma = 0$, is presented by the dashed lines in Fig. 1 [19].

From Fig. 1, it is easy to find that the DSKAW produces a local shocklike structure with a scale $\sim 10\lambda_e$ in the density distribution [see Fig. 1(a)] due to the dissipative effect of $\gamma \neq 0$. In particular, the nonsymmetrical distribution of the electric field [see Fig. 1(b)] can produce a net electric potential drop over the DSKAW. In consequence, the drop can accelerate the electrons to the order of the local Alfvén velocity v_A such that they escape finally by the downstream of the DSKAW [see Fig. 1(c)]. The density jump over the DSKAW, Δn, and the escaping velocity of the electrons, v_e, can be expressed in the parameter M_z as

$$\Delta n = 1 - M_z^{-2} \tag{14}$$

and

$$v_e = -(M_z^2 - 1) M_z v_A = -\Delta n (1-\Delta n)^{-3/2} v_A, \tag{15}$$

respectively. Figure 2 plots the escaping velocity $|v_e|$ versus the density jump Δn, where the dashed line represents the parallel phase speed of the wave M_z. For the solution of Fig. 1, one has the density jump $\Delta n = 40\%$ and the escaping velocity $v_e = -0.86 v_A$.
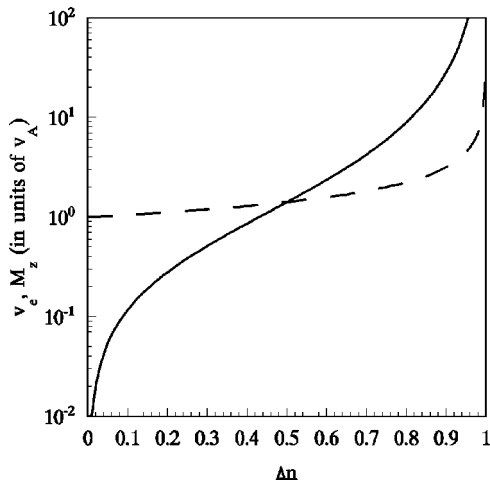
FIG. 2. Plot of the escaping velocity of electrons, $|v_e|$, against the density jump Δn of the DSKAWs, where the dashed line represents the parallel phase speed M_z.

From Fig. 2, one can find that the escaping velocity v_e has the magnitude $\sim(0.1-10)v_A$ for the variable range of $\Delta n \sim 0.1-0.9$. This indicates that the escaping electrons accelerated by the DSKAW can get energy of the order of $m_e v_A^2$ in most cases. The result suggests that the DSKAW could play an important role in the acceleration of the energetic electrons in astrophysical and space plasmas, since the Alfvénic fluctuation is the most common electromagnetic activity in cosmic plasmas. As an example, we will discuss below the possibility of applying the DSKAW to the acceleration of the Earth's auroral and solar coronal energetic electrons.

It has long been known that the discrete aurora is associated with energetic electrons with the energy in the range of $1-10$ keV, which impact the ionosphere [21]. The physical nature of the acceleration of these auroral energetic electrons, however, is still an open problem. It is interesting to note the fact that the local Alfvén velocity v_A varies along the auroral magnetic-field lines and reaches its maximum at the auroral acceleration region that is located in the altitude $\sim 5000-12\,000$ km above the auroral ionosphere [7,21]. For the acceleration region, the typical plasma parameters can be taken as $B_0 \simeq 0.06$ G and $n_0 \simeq 10$ cm^{-3}, and therefore the local Alfvén velocity $v_A \simeq 40\,000$ km/s. Consequently, the escaping electrons accelerated by the DSKAW from the acceleration region have a typical energy $\sim m_e v_A^2 \sim 10$ keV, which can meet the requirements of the acceleration of the observed auroral energetic electrons.

On the other hand, it has been widely accepted that the energetic electrons of a few tens of keV in the solar corona have been most directly detected through their gyromagnetic emission in the microwave and their bremsstrahlung emission in the hard x ray during solar flares [22–25]. Voitenko [26] demonstrated that the proton beams, which were set up by the reconnection outflow at the onset of a solar flare, could excite effectively the KAWs at a high growth rate ($\sim 10^5$ s^{-1}) and in a short relaxation distance (~ 10 km) in a flare loop. It can be expected that the DSKAW excited by the flare proton beams can possibly play an important role in the acceleration of the flare energetic electrons too. In fact, the typical parameters in the solar flare loops can be taken as $B \sim 10^3$ G, $n \sim 10^9$ cm^{-3}, and $T_e \sim 5 \times 10^6$ K~ 500 eV, and hence the pressure parameter $\beta \sim 2 \times 10^{-5} \sim 0.04 Q \ll Q$ and the local Alfvén velocity $v_A \sim 70\,000$ km/s. Consequently, the escaping electrons accelerated by the DSKAW in the flare loops have a typical energy of $\sim m_e v_A^2 \sim 30$ keV, which can meet the requirements of the acceleration of the flare energetic electrons [22–25].

In summary, in this paper we presented a model of nonlinear KAWs, called DSKAW (*dissipative* solitary kinetic Alfvén wave), in which the effect of the electron collisional dissipation has been taken into account. The result showed that the DSKAW produced a local shocklike structure with a net electric potential drop. In particular, we argue that the DSKAW can act as an efficient and common acceleration mechanism of the energetic electrons, and accelerate the electrons to the order of the local Alfvén velocity. The acceleration of energetic electrons has been one of the most outstanding problems in astrophysics, space physics, and plasma physics for a long time. Considering the fact that the Alfvénic fluctuation is the most common electromagnetic activity in extensive cosmic plasma environments, the present result suggests that the DSKAW can possibly play an important role in the acceleration of cosmic energetic electrons and in the energization of cosmic plasmas. Finally, as an example, we also discuss the possibility of applying the DSKAW to the acceleration of the Earth's auroral and solar flare energetic electrons.

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